Lösungsvorschlag Übungsvor Maschinenbau
SoSe 2009 A

1.) Pendel 1: $A = 10°$ $m = 20\,kg$ $\frac{T}{4} = 2\,sec$ $g = 9{,}81\,\frac{m}{s}$

$L = ?$ $\Rightarrow T = 8\,sec$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}} \Rightarrow \frac{4\pi^2}{T^2} = \frac{g}{L} \Rightarrow L = \frac{g T^2}{4\pi^2} =$$

$$\underline{15{,}9\,m}$$

2.) Pendel 2: $A = 10°$ $m = 20\,kg$ $T = 8\,sec$ $v_{max} = ?$
$= 0{,}17\,rad$

$$x(t) = A \cdot L \cdot \cos(\omega t)$$

$$v(t) = -AL \cdot \omega \cdot \sin(\omega t)$$

$$v_{max} = +A \cdot L \cdot \omega = \frac{A \cdot L \cdot 2\pi}{T} = \underline{2{,}1\,\frac{m}{s}}$$

3.) Pendel 3: (gedämpft) $A(t=0) = 10° := A_0$ $A(t_1 = 300s) = 7°$

$A(t_2 = 600\,sec) = ?$

$$A(t) = A_0 \cdot e^{-t/\tau}$$

$$\Rightarrow \frac{t_1}{\tau} = -\ln \frac{A(t_1)}{A_0} \Rightarrow \tau = \frac{t_1}{\ln \frac{A_0}{A(t_1)}} = 941\,sec$$

$$\Rightarrow A(t_2) = A_0 \cdot e^{-\frac{t_2}{\tau}} = \underline{4{,}9°}$$

4.) Gitterweite: $v_p = 600\,\frac{m}{s}$ $L = 1\,m$ f close to $f = 300\,Hz$

$$L = n \cdot \frac{\lambda_n}{2} \Rightarrow \lambda_n = \frac{2L}{n} \quad v_p = \lambda_n \cdot f_n \Rightarrow f_n = \frac{v_p}{\lambda_n}$$

$$= \frac{n \cdot v_p}{2L}$$

$\Rightarrow f_1 = 300\,Hz$ $\underline{f_2 = 600\,Hz}$ $f_3 = 900\,Hz$

5.) angeregte Saite $f_0 = 100\,Hz$ $f_1 = 100\,Hz$, $f_2 = 200\,Hz$, $f_3 = 300\,Hz$

$A_1 = 5\,cm$, $A_2 = 0{,}2\,cm$, $A_3 = ?$

$$A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_i^2)^2 + \frac{\omega_i^2}{\tau^2}}} \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{(\omega_0^2 - \omega_2^2)^2 + \frac{\omega_2^2}{\tau^2}}{\frac{\omega_1^2}{\tau^2}}}$$

$$= \frac{\sqrt{\tau^2(\omega_0^2 - \omega_2^2)^2 + \omega_2^2}}{\omega_1} \Rightarrow \tau = \sqrt{\left[\left(\frac{A_1}{A_2}\right)^2 - \left(\frac{\omega_2}{\omega_1}\right)^2\right] \cdot \frac{\omega_1^2}{(\omega_0^2 - \omega_2^2)^2}}$$

$$= \sqrt{\underbrace{\left(\frac{A_1}{A_2}\right)^2 - \left(\frac{\omega_2}{\omega_1}\right)^2}_{= 24{,}9}} \cdot \underbrace{\frac{\omega_1}{|\omega_0^2 - \omega_2^2|}}_{5{,}3 \cdot 10^{-4}} = 0{,}013\,s$$

$$A_3 = A_1 \cdot \frac{\frac{\omega_1}{\tau}}{\sqrt{(\omega_0^2 - \omega_3^2)^2 - \frac{\omega_3^2}{\tau^2}}} = A_1 \frac{\omega_1}{\sqrt{\underbrace{\tau^2(\omega_0^2 - \omega_3^2)^2 - \omega_3^2}_{1{,}7 \cdot 10^9\,1/s^2}}} = \underline{0{,}076\,cm}$$

6.) Kreiswelle: $d_1 = 10\,cm$

$A_1 = 5\,mm$

$d_2 = 30\,cm$ $A_2 = ?$ $A = \frac{\alpha}{\sqrt{d_i}} \rightarrow \frac{A_2}{A_1} = \sqrt{\frac{d_1}{d_2}}$

$$\Rightarrow A_2 = A_1 \cdot \sqrt{\frac{d_1}{d_2}} = \underline{2{,}9\,mm}$$

7.) Auto + 4 Insassen auf Wellenbahn: $\mu = 600\,kg \quad m = 80\,kg \Rightarrow \mu_{ges} = 920\,kg$

$\lambda = 10\,m \quad v_{max} = 20\,\frac{m}{s} \Rightarrow f_0 = \frac{v_{max}}{\lambda} = 2\,Hz$

$\omega = 2\pi f = \sqrt{\frac{D}{\mu_{ges}}} \Rightarrow D = \mu_{ges} \cdot 4\pi^2 \cdot f^2 = 145000\,\frac{N}{m}$

$F = D \cdot \Delta x \Rightarrow \Delta F = D \cdot \Delta x \Rightarrow \Delta x = \frac{\Delta m \cdot g}{D} = \frac{4 \cdot m}{D} = \frac{4mg}{D} = 0{,}02\,m = 2\,cm$

8.) Mond $\Delta x = 385.000\,km \quad v_p = 300.000\,\frac{km}{s}$

$\Rightarrow \Delta t = \frac{\Delta x}{v_p} = 1{,}28\,sec$

9.) Mikroskop: $D = 10\,cm \quad f_1 = 0{,}9\,cm \quad f_2 = 1{,}1\,cm$

$G_1 = 0{,}1\,cm \quad g_1 = 1\,cm$

$\frac{1}{f} = \frac{1}{g} + \frac{1}{b} \Rightarrow \frac{1}{b_1} = \frac{1}{f_1} - \frac{1}{g_1} = 0{,}11/cm \Rightarrow b_1 = 9{,}09\,cm$

$B_1 = G_1 \cdot \frac{b_1}{g_1} = 0{,}90\,cm = G_2$

$g_2 = 0{,}91\,cm \Rightarrow \frac{1}{b_2} = \frac{1}{f_2} - \frac{1}{g_2} = -0{,}11/cm \Rightarrow b_2 = -9{,}09\,cm$

$\Rightarrow B_2 = G_2 \cdot \frac{b_2}{g_2} = -0{,}9\,cm$

Bildgröße: $0{,}9\,cm$

10.) 2 Wellen auf Seil 1 $\quad A_1 = A_2 = 4\,cm$

$\varphi_1 = 0 \qquad \varphi_2 = \frac{\pi}{2}$

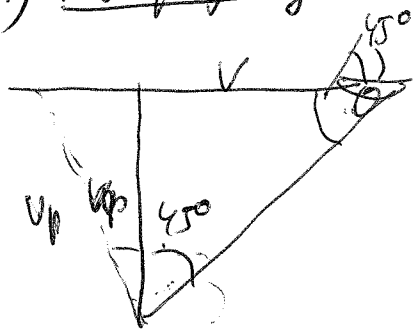
$$s_{ges}(x,t) = A_1 \left(\cos(\omega t - kx + 0) + \cos(\omega t - kx + \tfrac{\pi}{2}) \right)$$

$$= \underbrace{2A_1 \cdot \cos\tfrac{\pi}{4}} \cdot \cos(\omega t - kx + \tfrac{\pi}{4})$$

$$\Rightarrow A_{ges} = 2A_1 \cdot \cos\tfrac{\pi}{4} = \underline{\underline{5{,}66\,cm}}$$

11.) Düsenflugzeug: $\quad h = 5\,km \qquad \varphi = 45°$ (Überschallknall)

$v_p = 330\,\frac{m}{s}$



$$\sin\varphi = \frac{v_p}{v} \Rightarrow v = \frac{v_p}{\sin\varphi} = \underline{\underline{466\,\frac{m}{s}}}$$

12.) Bambusrohr anblasen: $\quad f_0 = 100\,Hz \qquad v_p = \lambda_0 f_0 \qquad \frac{\lambda_0}{2} = L$

f_1 (2. Ton) $= ?$

$$\Rightarrow L = \frac{v_p}{2 f_0}$$

$$L = \frac{v_p}{4 f_1} \qquad \frac{\lambda_1}{4} = L$$

$$\Rightarrow 1 = \frac{4 f_1}{2 f_0} \Rightarrow f_1 = \frac{f_0}{2} = \underline{\underline{50\,Hz}}$$
